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The Role of the Integrated Image of the Problem Solving Method in the Realization of the Mathematics Teaching Continuity

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Abstract: In the article we investigated the problem of forming in students' minds an integrated image of the way of solving mathematical problems. There were considered the possibilities of realization of intrasubject integration in the process of studying math. The methodical conditions under which due to the integrated image of the method of solving the problem the realization of the continuity of teaching mathematical disciplines during the transition from school to university was revealed. We have chosen the method of completing the square to describe the authors' methods of teaching math. Curricula and school textbooks currently in force in Ukraine were analyzed. We also considered universities' curricula of Mathematical Analysis. An experimental study with students on the formation of an integrated image of the method of solving problems, which showed positive dynamics and proved the effectiveness of the developed methodology. It was concluded that the formation of an integrated image of the method of solving problems during a long study of mathematics was possible with the correct choice of problems that are solved in a given way; application of the system of didactic principles, in particular systematicity, consistency and perspective; constant generalization and systematization of knowledge, skills and abilities of students.

Keywords: *Methods of teaching math; integrated image; completing the square; methodical conditions; teaching continuity.*

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Introduction

One of the most important components of the New Ukrainian School pedagogy is integrated learning, which indicates its relevance, modernity, necessity and effectiveness. Integrated learning is a process during which connections between concepts and experience are established so that previously acquired knowledge and skills can be applied to new and more complex tasks or problems. This definition is closely intertwined with the definition of the mathematical disciplines continuity principle, given in (Botuzova, 2021), which is a didactic principle aimed at providing students with the necessary level of mathematical training, which will allow them to continue studying mathematical disciplines at higher levels of education; provides the extrapolation of goals, content, forms, methods and tools of teaching mathematics, as well as the integration of related disciplines, establishing interdisciplinary links, the implementation of practical orientation and professional orientation of teaching mathematics in the educational process at related educational levels.

The concept of integration in teaching is a multi-aspect phenomenon. In her research, V. Kryshmarel (2021) analyzed the essential content of the "integration" concept in the context of general education taking into account the world practice and the work of Ukrainian scientists. For example, she pointed out the relevance and possibilities of implementing interdisciplinary and intradisciplinary integration in social sciences teaching.

A group of researchers from Thailand (Kaewsaiha et al., 2016) demonstrated the possibility of using integrated teaching strategies to improve mathematics and science education in reforming the teaching profession.

When studying mathematics, it is possible to carry out both intra-subject integration, when connections are established between the blocks of studying educational material within one subject, and interdisciplinary integration, where such links are established between different disciplines. For instance, recommendations for improving interdisciplinary integration in mathematics lessons are given in the contributions of V. Gogovska, R. and Malcheski (2012). P. Treacy and J. O`Donoghue (2013) offered a customized pattern of interdisciplinary integration of mathematics and science at school called "Authentic Integration".

V. Kushnir and R. Rizhniak (2011) in their study of the integrative approach in teaching mathematics determined the integrated images of the problem, the problem topic, the method of proving the mathematical statement, the method of solving the problem. Among the latest studies, the article (Akbash et al., 2021) on the possibilities of implementing an integrative approach using integrated images, specifically an integrated image of the problem topic stands out.

In the current study we will deal with the interpretation of the integrated image of solving mathematical problems method proposed in the article (Kushnir & Rizhniak, 2010) where it is defined as a holistic structure of knowledge, skills and abilities that students must have to assess the possibility of selected problem solving method.

Methodology of Research

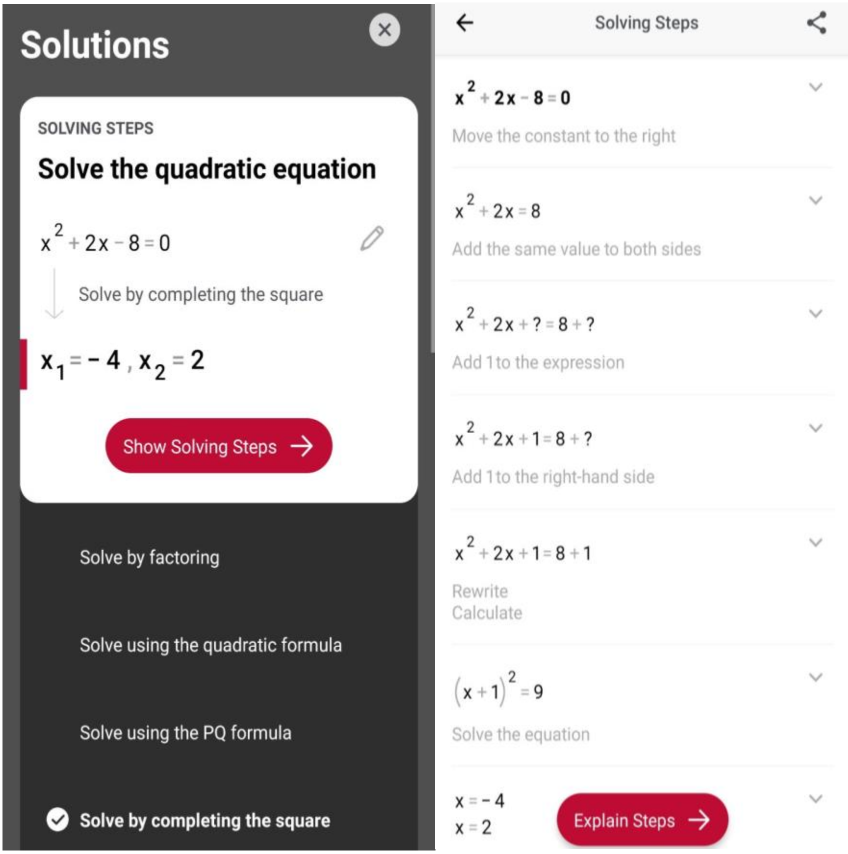
The aim of the article is to establish the role of an integrated image of the problem solving method in the implementation of mathematics teaching continuity on the example of completing the square method.

In this article we will try to detail and reveal the methodological conditions under which, thanks to the integrated image of the problem solving method, the continuity of teaching mathematical disciplines is realized vertically during the transition from school to higher educational establishments.

The reason for starting this study was the observation, in particular, that there is usually revision of the educational material studied in previous school years at the first algebra lessons in the 9th grade. Considering the material laid down in the 9th grade mathematics curriculum, and referring to the principle of continuity, teachers should make sure that students have mastered such topics as: "Formulas of abridged multiplication", "Quadratic equations". It is appropriate to do student assessment for this purpose.

Owing to the research carried out for several years in a row, sufficient empirical material has been accumulated, where, along with the quantitative characteristics observations and analysis of approaches to the tests done by students stir interest. The way to solve complete quadratic equations is of special interest.

Analyzing the mathematics curriculum for the 8th grade and relying on the own pedagogical experience of many years and the experience of colleagues, we can predict that students will use the method of finding the discriminant and the quadratic formula. However, we managed to establish the following fact: every year, starting from 2017-2018 academic year 1 or 2 from 30-34 students in a class solve the complete quadratic equation by completing the perfect square of the binomial. This method is necessary for further study of mathematics in the 9th grade, so it is important that all students in the class master it. On the other hand, a more detailed study of this fact shows that even a small percentage of students who have used the method of completing the square to solve quadratic equations cannot reproduce it in another context, or cannot explain the solution at all. The problem, as it turned out, was that these students used the popular mobile application PhotoMath, which scans and recognizes the equation, gives its roots and offers to review the detailed steps of the solution process with explanations for each of them. To illustrate, we will show how the PhotoMath application works in the process of solving a complete quadratic equation, for example (Fig. 1).



(a) (b)

Figure 1. Illustration of how the mobile application PhotoMath works: a) the condition of the problem and the obtained roots of the equation with the proposal to choose the method of getting the obtained roots; b) detailed steps of solving the equation.
Source: Authors' own conception

In Fig. 1 a) this mobile application offers several ways to solve the complete quadratic equation, including factoring with the help of the roots of the quadratic equation formula, using the PQ formula (works only for the consolidated quadratic equation), by completing the square.

At the same time, Fig. 1 b) presents all the steps for solving the quadratic equation by completing a square, and provides an opportunity to explain in detail all the steps by clicking on the button "Explain the steps".

Analyzing all textbooks on algebra for the 8th grade recommended by the Ministry of Education and Science of Ukraine, we state that only in one of them (Tarasenkova et al., 2016) a special place in the form of a paragraph is given to solving quadratic equations by completing the square of a binomial, two other textbooks (Bevz & Bevz, 2016; Prokopenko et al., 2016) give examples of solving quadratic equations by completing a square; based on this method the same paragraph displays the formula for the roots of a quadratic equation, at the same time the textbook Prokopenko, Zakhariichenko & Kinashchuk (2016) states that “Solving a quadratic equation by completing a square contains tedious conversions and is not suitable, especially if the second coefficient is an odd number”. A similar approach is used in the textbook (Mal′ovanyy et al., 2016), but the authors first derive the root formula for the consolidated quadratic equation (PQ-formula), and then share the obtained result to get the formula for the roots of the equation such as $ax^2+bx+c=0$. The rest of the textbooks (Easter, 2016; Kravchuk et al., 2016; Merzlyak et al., 2016) immediately offer students the derivation of the formula for the roots of a quadratic equation by the synthetic method, such as: multiply both parts of the equation $ax^2+bx+c=0$ by $4a\neq 0$ and, obtaining an equation $4a^2x^2+4abx+4ac=0$ equivalent to this, complete the perfect square in its left part. The artificiality of this method of presenting educational material is obvious and as the result the students follow the plan of the formula proof (conclusion) proposed by the textbook or the teacher. If the students’ activities were analytical and synthetic in nature, then awareness would be provided in the implementation of the proof plan.

Results and discussion

We will try to outline the importance of students mastering the method of completing a square, based on the principle of continuity and forming an integrated image of solving problems method, starting with the algebra of the 7th grade of a secondary school and ending with mathematics in universities.

According to the current curriculum in mathematics for the 7th grade algebra, the topic "Integer Expressions" is studied and one of the expected results of it is that the student solves exercises that involve the use of formulas for abbreviated multiplication in the process of solving equations and proof of statements. For successful mastering of educational material, it is important to choose an appropriate system of mathematical exercises, which should be, according to Erdniev & Erdniev (1986), complete. For this, in addition to exercises such as "Collapse this trinomial in the square of the two monomials difference" or "Submit a trinomial in the form of a square of a binomial", students should be offered so-called "altered exercises" (Erdniev & Erdniev, 1986) such as: “Replace asterisks with such monomials which give identity". For example, we compare the following exercises and mental effort required to solve each of them: 1) give a trinomial $4x^2+25y^2-20xy$ in the form of a square of a binomial (Bevz & Bevz, 2015, p. 97); 2) which monomial should be placed instead of an asterisk in the expression $9c^2-12c+*$ so that it can be represented as a square of a binomial (Merzlyak et al., 2017a, p. 109).

Obviously, the solution of exercise 2) is based on finding the missing element. This makes the thinking process more complex and meaningful compared to what happens during exercise 1), and therefore it has a better developmental effect for students. Exercises of type 2) - the so-called "reverse tasks" - naturally develop skills of self-control, which is carried out involuntarily or even subconsciously, while the solving of exercises type 1), the so-called "direct structure tasks", ends only by obtaining answers, and control and verification are done only at the teacher’s request.

Algebra textbooks for 7th grade of SLEI (secondary level education institution) offer propaedeutic exercises on the topic "Quadratic equations", for example: 1) Solve the equation $z^2+14z+49=0$ (Bevz & Bevz, 2015, p. 117); 2) Prove that there are no roots of the equation $x^2-14x+52=0$ (Merzlyak et al., 2020, p. 126).

It should be mentioned that the content of students’ integrative learning activities in solving problems with the help of a pre-selected way, looks like this (Kushnir & Rizhniak, 2010): the formation of an integrated image of the solution, which is a holistic structure of knowledge, skills and abilities that are a condition for mastering this method of solving problems; analysis and comparison of the content components features of the solution integrated method; abstraction from insignificant features of its components; generalization and classification of components of the integrated image; systematization, which is the further distribution and unification not of the individual components of the integrated image of the solving a group of problems method, but of their classes; synthesis of new knowledge. We will show below how it happens on an example of completing a perfect square method.

Among the tasks that correspond to the high level of students achievement, in algebra textbooks for the 7th grade there are also propaedeutic exercises to prove inequalities, for example: Exercise 1. Compare with zero the value of the expression a) x^2-4x+4; b) $-x^2+2x-1$ (Easter, 2015, p. 92); Exercise 2. Prove that this expression takes positive/negative values for all values of x. Specify what is the smallest/largest value of this expression and at what value of x: $16x^2+24x+25\,/-x^2+4x-12$ (Merzlyak et al., 2020, p. 126). Performing such tasks requires more mental effort from students. The experience gained by students in performing simpler tasks will allow them to notice in expressions a) x^2-4x+4 and b) $-x^2+2x-1=-\left(x^2-2x+1\right)$ perfect squares: a) $(x-2)^2$; b) $-(x-1)^2$. This, in its turn, will help to evaluate the values of the expressions and compare them with zero, using the properties of the square number. In the case of proving that this expression takes only positive/negative values for any value of the variable (exercise 2) the method of completing the square is also directly used.

It should be mentioned that the exercises identical to Exercise 2 are given in all textbooks of algebra for the 9th grade of SLEI in the topic "Inequalities", but before doing them the 9th grade students should study the properties of numerical inequalities (by assessment method (Botuzova, 2020). But practical experience with 9th graders shows that it is quite difficult for students to master the material related to proving inequalities by completing a square, especially when the coefficient near the expected "twice the product" is an odd number. This fact in particular initiated the work on the presented publication.

After analyzing the current textbooks on algebra for 7- 9 grades it is possible to state that there is a system of exercises in them which allows students to master well a way of completing a square. At the same time, violation of the teaching mathematics continuity principle (Botuzova, 2021) which becomes obvious in the absence of the above mentioned solving problems method repetition with other conditions, at a higher level (especially in 8th grade while studying quadratic equations) causes a problematic situation in the 9th grade.

Considering the applicability of the integrated image of the completing a square method in the further study of mathematical disciplines (what will be discussed below), it is still better to refer to it in the 8th grade, which will ensure the continuity of teaching.

As for the applicability of the integrated image of the above-mentioned method, in the 9th grade it is used both in the course of algebra and in the course of geometry and it can be seen later in mathematical disciplines studied by students at universities. We will give examples from current textbooks and outline the appropriate ways to implement the principle of continuity at the horizontal and vertical levels.

Examples (grade 9, algebra): "Prove the inequality a) $4a^2+1\geq 4a$, where a is any number (Tarasenkova et al., 2017, p. 48); b) $2a^2+4a+5>0$ (Bevz & Bevz, 2017, p. 63); c) $a^2+b^2+6a-4b+13\geq 0$ (Merzlyak et al., 2017a, p. 11); d) $5a^2+4a-2ab+b^2+2>0$ (Kravchuk et al., 2017, p. 12); e) $m^2+n^2+1\geq m+n+mn$ (Easter, 2017a, p. 11)».

Selected from the current textbooks of algebra, the system of exercises shows the possibilities of applying the method of completing a perfect square to prove inequalities, taking into account that $x^2\geq 0$ for any value of x, $x^2+a>0$ provided that $a>0$, x is any real number, as well as using the properties of numerical inequalities. Thus, for example: a) it is enough to perform simple transformations $4a^2+1-4a\geq 0$ and show that the expression obtained in the left part of the inequality is a perfect square of the binomial $(2a-1)^2\geq 0$ and the inequality is proved. In example b) you should put a constant factor out of parentheses $2\left(a^2+2a+\frac{5}{2}\right)>0$ and complete the perfect square in parentheses by decomposing the number by the terms $\frac{5}{2}=1+\frac{3}{2}$. Then we have $2\left(\left(a^2+2a+1\right)+\frac{3}{2}\right)>0$. Thus, it turns out that $2(a+1)^2+3>0$ - the statement is true. More complex transformations based on regroupings are performed during the solving of example c) $a^2+b^2+6a-4b+13\geq 0$; $\Rightarrow$ $\left(a^2+6a\right)+\left(b^2-4b\right)+13\geq 0$; $\Rightarrow$ decomposing the number 13 into terms 9 and 4, we obtain: $\left(a^2+6a+9\right)+\left(b^2-4b+4\right)\geq 0$; $\Rightarrow$ where $(a+3)^2+(b-2)^2\geq 0$, which proves the given inequality.

Solving examples d) and e) require experience in solving simpler examples, such as a) -c) and using a more powerful logic apparatus. In particular, example d) $5a^2+4a-2ab+b^2+2>0$ requires prior visual study and analysis. The presence of a twice product of $2ab$ indicates that you can compete the square of a binomial with two variables, for example:

$a^2 - 2ab + b^2$, then the left side of the inequality is rewritten as follows: $a^2 + 4a^2 + 4a - 2ab + b^2 + 2 > 0$; $\Rightarrow$ $\left(a^2 - 2ab + b^2\right) + \left(4a^2 + 4a + 1\right) + 1 > 0$; $\Rightarrow$ $\left(a-b\right)^2 + \left(2a+1\right)^2 + 1 > 0$. The transformations performed prove the inequality for any real value of the variables.

Example e) - is a task of increased complexity, especially because when transferring all terms to the left side of the inequality, the presence of a perfect square does not become obvious: $m^2 + n^2 + 1 - m - n - mn \geq 0$. Complex manipulations allow you to rewrite the left side of a given inequality in the form: $\left(\frac{m}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{n}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{m}{\sqrt{2}} - \frac{n}{\sqrt{2}}\right)^2 \geq 0$. The final form of the left side of the inequality is the sum of three squares of the two terms, which is the number of non-negative variables for any value and the inequality is proved.

In the same textbooks, the method of completing the square of a binomial is used in the study of the functions graphs transformations in the topic "Quadratic Function". By the way, in the textbook (Easter, 2017a, p. 97) exercises for revision are offered: "Complete the square of a binomial from a square trinomial: $x^2 + 6x - 2$".

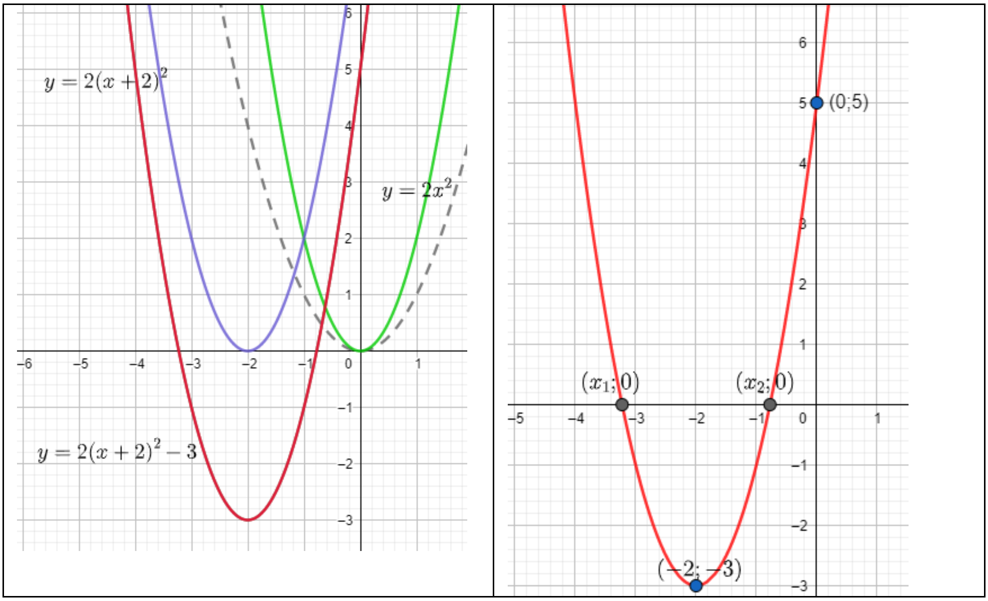
By the 9th grade, students have already accumulated a significant amount of knowledge about the graphs of functions: $y = kx + b$ - line (the 7th grade); $y = \frac{k}{x}$ - hyperbola, $y = x^2$ - parabola, $y = \sqrt{x}$ - parabola branch (the 8th grade). In the 9th grade they get acquainted with a way of plotting graphs of functions by means of geometrical transformations, in particular parallel transfer along coordinate axes, and compression or stretching. For a quadratic function $y = ax^2 + bx + c$, $a \neq 0$ it is proposed to use the method of completing a perfect square, as a result of which the analytical form of the function is rewritten as follows: $y = a\left(x \pm m\right)^2 \pm n$, where $m, n \geq 0$, which allows to construct a parabola by several successive transformations.

Further the method of plotting a quadratic function is studied $y = ax^2 + bx + c$, $a \neq 0$ according to the algorithm, when the direction of the parabola branches, the coordinates of its vertex, the points of intersection with the coordinate axes, etc. are established. Thus in textbooks on algebra by means of completing a square of a binomial in the general way for the expression $ax^2 + bx + c$ defining quadratic function, the explanation, concerning expediency of application of a parabola construction algorithm, which provides search of coordinates of its top, is carried out $x_v = \frac{-b}{2a}$, $y_v = \frac{4ac-b^2}{4a} = \frac{-D}{4a}$, where D is the discriminant or $y_v = ax_v^2 + bx_v + c$.

Let us consider an example of plotting a quadratic function in both of the above ways.

Table 1. Example. "Build a graph of the function $y = 2x^2 + 8x + 5$". Comparison table of methods

Using successive geometric transformations (completing a square)	Using the algorithm (finding the coordinates of the vertex)
Complete the square of the binomial in the expression $2x^2+8x+5$: $2\left(x^2+4x+\frac{5}{2}\right) = 2\left(\left(x^2+4x+4\right)-4+\frac{5}{2}\right) =$ $= 2\left(\left(x+2\right)^2-\frac{3}{2}\right) = 2\left(x+2\right)^2-3$. Thus, to plot the graph of function $y=2x^2+8x+5$, you must perform a series of successive transformations with the graph $y=x^2$: $y=x^2$ $\Rightarrow$ $y=2x^2$ (stretching twice along the axis OY) $\Rightarrow$ $y=2\left(x+2\right)^2$ (moving 2 units to the left along the axis OX) $\Rightarrow$ (moving 3 units down along the axis OY).	Let us follow the algorithm of construction: $a=2>0$ - branches up; $x_v = \frac{-8}{2\cdot 2} = -2$; $y_v = 2\cdot(-2)^2+8\cdot(-2)+5=-3 \Rightarrow$ $(-2;-3)$ - coordinates of the parabola vertex; the point of intersection with the axis OY: $x=0 \Rightarrow y=2\cdot 0^2+8\cdot 0+5=5$ - $(0;5)$; points of intersection with the axis OX: $y=0 \Rightarrow 2x^2+8x+5=0$; $x_1=-2-\frac{\sqrt{6}}{2}$; $x_2=-2+\frac{\sqrt{6}}{2}$ (for the obtained $x_{1,2}$ it is necessary to determine the approximate values). $x_1 \approx -3,2$; $x_2 \approx -0,8$. Let us put all the defined points on the coordinate plane.



Source: Authors' own conception

Taking into account the peculiarities of the modern students generation, in order to develop their intellectual potential, after getting acquainted with both methods of plotting a quadratic function, it is useful to create at least one such construction in both ways together with comparative analysis, which would be carried out by students interactively.

Moreover, updating of the competing a square method is necessary for the course of geometry for the 9th grade, which studies the equation of a circle and there are problems of this type: «Find the radius and center of the circle given by the equation $x^2+y^2+4x-10y-7=0$ (Easter, 2017b, p. 33)», «Determine whether this equation is a circle equation. In the case of an affirmative answer, indicate the coordinates of the center and the radius of this circle $x^2+y^2+6y+8x+34=0$ (Merzlyak et al., 2017a, p. 88)». Solving such problems involves the reduction of given equations to the general form, which includes squares of binomials: $(x-a)^2+(y-b)^2=R^2$, where $(a;b)$ - the coordinates of the center of the circle, R - its radius.

There are similar type of exercises in textbooks in geometry for the 10th grade, intended STEM majors level, when studying the equation of a sphere in the topic "Coordinates and vectors in space": «Find the coordinates of the center and the radius of the sphere $x^2+y^2+z^2-16y+6z=0$ (Merzlyak et al., 2017b; 2018, p. 196]», «Prove that the equation $x^2+y^2+z^2-4x+2y-4=0$ is the equation of the sphere. Find the center of the sphere and its radius (Easter & Ergina, 2018, p. 305)». This allows teaching continuity, which must establish links within and between levels of education for further development of the competencies, formed in students on the previous stages, which involves the usage of a didactic principles system, including regularity, consistency and potential, regulatory support, and also requires constant interaction of the learning process subjects for the personal development of students in the process of adaptation to changes in learning conditions (Botuzova, 2021). At the same time, at connected levels of education, in particular when moving from school to university, the principle of continuity of mathematics disciplines is aimed at providing students with the necessary level of mathematical background, which will allow them to continue studying mathematics at higher levels of education.

As it was mentioned before, the method of completing the square is used in studying the general course of higher mathematics in universities, especially in the section «Integral calculus»: *Exercise 1*. Find the integrals: a) $\int \frac{dx}{3x^2-8x-3}$; b) $\int \frac{(x+5)dx}{\sqrt{3-6x-x^2}}$ (integration of expressions containing a quadratic trinomial) (Botuzova et al., 2019).

In the course of analytical geometry studying the topic «Quadratic Forms» the problems of quadratic forms reduction to the canonical form are solved. One of the simplest methods of performing such tasks is to complete a square - the Lagrange Method. For example, *Exercise 2*. Bring the quadratic form $x_1^2+2x_1x_2+2x_2^2+4x_2x_3+8x_3^2$ to the canonical form (Yaremenko & Lutchenko, 2005).

During the research, we planned and conducted a pedagogical experiment. It involved using an integrated image of solving mathematical problems method when teaching mathematics to the 9th grade students. The developed technique required creation of conditions for students to master the method of completing the square when solving algebraic and geometric problems. The need to provide: individual approach to students by developing several similar task options; differentiation of training due to the creation of different complexity level tasks; elements of heuristic learning by creating problem situations, the content of which was to find a way to solve a problem that has not been encountered before; generalization of knowledge systematization and skills of students was taken into account.

We chose 258 9th grade students from two schools in Kropyvnytskyi (Ukraine) for the experiment. Age of participants was 14-15 years old. The distribution of participants in the experiment is presented in table 2.

Table 2. The distribution of participants in the pedagogical experiment

Year of the experiment	Experimental Group		Control Group		Total
	Class 9-1	Class 9-2	Class 9-A	Class 9-B	
2018/2019	30	32	32	34	128
2019/2020	34	32	32	32	130
Total	128		130		258

Source: Authors' own conception

Preliminary, with the help of diagnostic control work, which was discussed earlier, it was found (by criterion χ^2) that the mathematics knowledge level of the students of the experimental and control groups does not have significant difference statistically. Training is carried on according to one curriculum approved by the Ministry of Education and Science of Ukraine. The experiment in no way violates the curriculum, but is only an attempt to improve approaches to teaching mathematics properly.

The educational experiment with each experimental class was conducted during one academic year. Students of control classes studied in the usual mode.

At the end of the experiment all respondents were offered a test, which consisted of different complexity level tasks: 1) tasks, in which it was indicated how to solve the problem; 2) tasks that do not specify the method of solution, but they are familiar to students; 3) unfamiliar to students tasks, which do not specify the method of solving the problem. The test was marked on a 12-point scale. We were able to classify the results obtained by students by levels: high (12-10 points), sufficient (9-7 points); average (6-4 points); low (3-1 points). The results of the final test are listed in table 3 and are presented in Fig. 2.

Table 3. The experiment participants' results of the test

Level	Experimental Group		Control Group	
	Number of students	% of students	Number of students	% of students
High	23	17,97	9	6,93
Sufficient	60	46,88	54	41,54
Average	35	27,34	46	35,38
Low	10	7,81	21	16,15

Total	128	100	130	100

Source: Authors' own conception

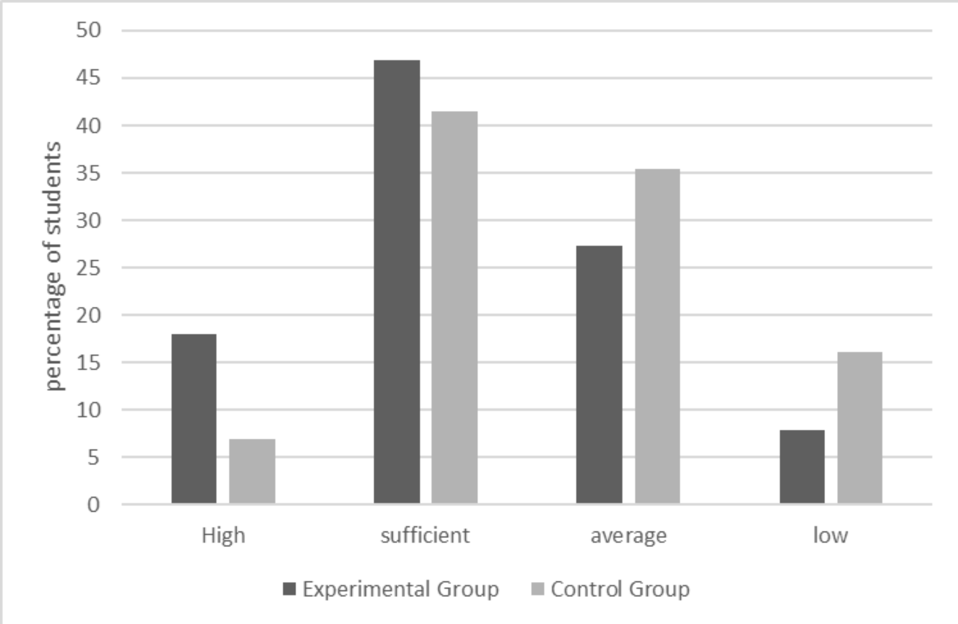


Figure 2. Comparison graph of the test results
Source: Authors' own conception

Statistical hypotheses at the final stage of the experiment are formulated as follows: H_0 - the distribution of students' levels of mathematical knowledge and skills in solving problems of the EG and CG participants at the final stage of the experiment do not differ significantly; H_1 - the distribution of students' levels of formation mathematical knowledge and skills in solving problems of the EG and CG participants at the final stage of the experiment differ significantly.

Pearson criterion χ^2 was used to estimate statistical significance of obtained experimental results and find out the difference between groups of students at the final stage of the experiment. According to table 2, in particular, the empirical value of the criterion was calculated $\chi_e^2 \approx 30.837$. Comparing it with the critical value $\chi_c^2 = 7.815$ at the significance level of 0.05, we conclude that the differences in the distributions of the experimental and control groups are statistically significant, that is why the null hypothesis H_0 is rejected and the alternative hypothesis H_1 is accepted. The obtained results prove the effectiveness of the proposed method.

Conclusion

Returning to the research objective, we note that the methodological conditions for implementing the principle of teaching mathematics continuity through an integrated image of the solving problems method are: the formation of a holistic integrated image of the solving problems method during long-term mathematics study from school to higher education; selection of a set of problems that are solved in a given way (the choice of method can be predetermined or determined by heuristic activity); application of the didactic principles system, such as systematicity, consistency and perspective; constant generalization and systematization of knowledge, skills and abilities of students.

Thus, the presented material demonstrates the applicability of the integrated image method of a completing the square during the study of mathematics, the importance of mastering it by students and the possibility of implementing continuity in the study of mathematical disciplines at the horizontal and vertical levels. The study confirms the importance of using an integrated image of the solving problems method in the process of ensuring the continuity of mathematics teaching, which requires careful planning.

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