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The Implementation of an Integrative Approach to Learning with the Use of Integrated Images

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Abstract: The article is dedicated to the definition of methodological conditions under which the use of the solution and research of the "task series" generated by a given task topic will acquire methodological expediency in the context of developing students' abilities to solve mathematical tasks of a productive nature. The authors suggested a possible way to solve it through the synthesis of integrated knowledge and the formation of the integrated images of the task topics that allowed students to develop knowledge and skills of integrative mathematical activity.

Keywords: *methodical conditions, integrated image, task topic, task series, integrative mathematical activity.*

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Introduction

In recent years, the decline of the level of the students' motivation to the traditional forms of mastering educational material has been a serious problem of the Ukrainian school. In the presented study, we tried to figure out to what extent the new forms of the organization of their activities related to the research training motivate students' learning activities. This will be done by the example of disclosing the contents of various levels of integration in the process of solving mathematical problems. By the content of the integration level, we understand the volume of mathematical material used in solving a mathematical problem. Among the integration levels we will distinguish the following: integration within the framework of the topic; integration within the educational material section; integration within the discipline; integration within the framework of mathematical disciplines; integration between different disciplines.

In order for the search and creation of a way to solve problems to take place according to a certain plan, we will guide students to master the main ways (methods) of solution, among which the following main ones can be distinguished: dividing the task into sub-tasks; task transformation; coding of task objects. That is, we will take into account that the main method for solving mathematical problems is mathematical modeling. The type and nature of modeling is determined mainly by the nature of the student's abilities and skills to operate the material, the heuristic schemes for finding a solution, and the nature of the problem itself. In the following presentation, we will understand the mathematical model as a special description of a certain problem, situation, which allows us to use the formal logical apparatus of mathematics in the process of its analysis. In mathematical modeling, we are dealing with a sign system (for example, in the form of a graphic diagram), which in mathematical form expresses the basic laws and properties of the object under study. In the process of mathematical modeling, we will distinguish three stages 1) formalization - translation of the proposed problem (situation) into the language of mathematical theory (building a mathematical model of the problem) 2) solution of the problem inside the mathematical model, the result of which will be either a new model of the problem or the final answer; 3) translation of the result of the mathematical solution of the problem into the language in which the problem was formulated (interpretation of the obtained mathematical solution). So, by the process of solving a mathematical problem we mean the process of sequentially constructing new models of the problem situation of a particular problem, and each new model of the problem will have less uncertainty than the previous one.

The main problem of our research is to determine the methodological conditions under which the study of the "task series" generated by a given task topic will acquire methodological feasibility in the context of developing students' skills to implement the integrative activity in the productive operation of the mathematical material.

As the main focus of our study, we chose to teach Mathematics to the students of middle and senior school age (15-17 years) using the integrative approach. Based on research of Gritsenko (2008), Ekzhanova & Reznikova (2008), Kozlovskaya (1999) it includes: first, the definition of the objective prerequisites for combining the previously disparate content elements from a course in Mathematics; secondly, the combination of such elements is not a simple addition, but the result of synthesis; thirdly, obtaining the result of combining in the form of a system that has the properties of integrity.

The problems of integration of Mathematics in teaching activity are the topic of the research by the scientists from different countries. Developing a model for integrated teaching of Mathematics and environmental education was covered in Cotič et al. (2015), the relationship between problem solving skills, metacognitive awareness and Mathematics achievement was defined, and the role of metacognitive awareness as a mediator in Nurulhuda was clarified in Hassan & Rahman (2017), the concept of interdisciplinary integration in Sicherl Kafol (2007) was defined, the integrated integrated teaching of Mathematics in elementary school in Skupnjak (2009) was analyzed, the relationships between numerical content and algebraic thinking in Somasundram et al. (2019) were investigated. They concluded that the implementation of an integrated approach to the use of computer technology in the formation of abilities to teach mathematics helps to increase the pedagogical competence of primary school teachers and knowledge of mathematical material. Farzam & Allahdadi (2018) determined that an integrative approach to the study of mathematics in primary school students using educational games is an effective way to improve the quality of children's learning. Pehoiu (2019) emphasizes the importance of integrated education to create the right attitude, responsibility and motivation in environmental issues.

In the works of Rizhniak (2009a) and Rizhniak (2009b), the author comes to the conclusion that the integrative line in the school Mathematics course gradually finds a more detailed implementation in the use of educational mathematical tasks of integrative content. These are the tasks of a creative nature; the tasks with a wide mathematical content and a complex structure of the interrelations between the components of their plot; the tasks that have the potential to create the new tasks and a series of tasks on their basis. Their combination in the context of compiling new tasks can form a wide field of possibilities with a sufficiently high uncertainty. Mathematical problems with high uncertainty, in turn, expand the students' ideas about the various approaches, ideas, methods that provide their solutions. It encourages students to put forward and substantiate certain assumptions, build fragmented theoretical generalizations, thus ensuring the development of the process of forming creative, heuristic thinking in students, as well as the desire for research. The solvation of such problems requires the subjects to learn in-depth knowledge and creativity. Here, not only the knowledge of students on a particular topic is used, but also the need arises to systematize and summarize the knowledge gained from different sections of school Mathematics and other academic disciplines. It requires the formation of the subject of training of the appropriate level of mathematical and informational culture. In the work of Rizhniak (2010) on the example of using various ways to solve one problem, the author concluded that the students form relevant integrative knowledge and skills in the form of an integrated image of the task. By thus we understand the holistic structure of skills that a student (subject of study) needs to have in order to study the problem in terms of its solutions and study.

Though, given the general availability of theoretical studies on the implementation of an integrative approach to teaching, international research needs a methodical development of an experimental nature that assesses methodological expediency in the context of developing students' ability to carry out integrative activity in the productive operation of mathematical material.

Methodology of Research

We have developed the main prerequisites for the research in the form of the main stages of an experimental study: a) the determination of the indicators for evaluating the effectiveness of the method of forming integrative knowledge and skills among students when operating the mathematical material; b) the evaluation of the effectiveness of the traditional method of forming students' integrative knowledge and skills when operating the mathematical material; c) experimental verification of the effectiveness of the developed methodology.

During the experimental study, the following methods were used: theoretical: analysis of the psychological and pedagogical literature on the problem of the research; empirical: pedagogical observation of the educational and cognitive activity of the students, conversations with teachers of Mathematics; the mathematical methods of the statistical processing of experimental data (two-sample t-test for independent student samples (Glass & Stanley, 1976), that were used to determine the quantitative and qualitative dependencies between the research indicators.

The main requirements for conducting the experimental research were to ensure the reliability, validity and veracity of the experimental data. The experimental study used a standardized methodology for conducting a pedagogical experiment, which ensured the reliability of the experimental data. Reliability was also ensured by the selection of the highly qualified expert teachers of the general educational institutions where the pedagogical experiment was conducted.

The implementation of the pedagogical experiment under the conditions of a real educational process and the equalization of the conditions for its conduction for the control and experimental groups ensured the reliability of the experimental data obtained.

Results of Research

In the article, the authors intend to consider in detail the process of working on the task topic, where the approach to the formulation of the conditions of a specific task will vary depending on the level (reproductive, productive, creative) of skills required to solve the task (classes of mathematical competence). We explore the methodological feasibility of using the compilation and solution of the various tasks generated by a given task topic in the formation of the students' integrative knowledge and skills of operating the mathematical material.

By the integrated image of the task topic, we mean the holistic structure of knowledge and skills that a student (the subject of the study) needs to own in order to study the "task series" generated by the task topic for their solution and research. It should be noted that just as it is impossible to talk about the necessity of organizing the solution by all the subjects of learning all the tasks from the "task series" generated by the task topic, there is no point in talking about the largest (most complete) volume of the mentioned "task series". The volume of the integrated image of the task topic will be determined in accordance with the set goals of the educational activity.

Let us consider in more detail the ideas expressed by the example of a task theme, that is defined as follows: inscribing combinations of other geometric figures into specified geometric figures. We illustrate everything with specific examples.

<u>*Task 1.*</u> *In a square with side* a*, a circle was inscribed, a square in this circle, a circle in this square, etc. Find the ratio of the sum of the areas of all squares (circles) to the area of the largest square (circle) (or: find the ratio of the sum of the areas of all squares to the sum of the areas of all circles).*

To solve the task, it is necessary to find a pattern by which the areas of the geometric figures specified in the task change. It is obvious that the area of the largest square will be calculated by the formula: $S_1 = a^2$, and the area of the circle inscribed in it: $Q_1 = \frac{\pi a^2}{4}$ (since the side of the largest square will be the diameter of the circle inscribed in it). Since the radius of this circle will be the diagonal of the square inscribed in it, the side of this square will be $\frac{\sqrt{2}}{2}a$, therefore its area is: $S_2 = \frac{a^2}{2}$, and the area of the circle inscribed in it: $Q_2 = \frac{\pi a^2}{8}$. Similarly, we find: $S_3 = \frac{a^2}{4}, S_4 = \frac{a^2}{8}$, $Q_3 = \frac{\pi a^2}{16}, Q_4 = \frac{\pi a^2}{32}$. Thus, we obtained two sequences of areas. For squares:

$$a^2; \frac{a^2}{2}; \frac{a^2}{4}; \frac{a^2}{8}; \dots;$$

and for circles:

$$\frac{\pi a^2}{4}; \frac{\pi a^2}{8}; \frac{\pi a^2}{16}; \frac{\pi a^2}{32}; \dots$$

Both sequences are infinitely decreasing geometric progressions with a denominator $\frac{1}{2}$. The sum of the areas of all squares and the areas of all the circles will find the formula for the sum of the members of an infinite geometric progression with the denominator $|q| < 1$:

$$S = \frac{a^2}{1 - \frac{1}{2}} = 2a^2;$$

$$Q = \frac{\frac{\pi a^2}{4}}{1 - \frac{1}{2}} = \frac{\pi a^2}{2}.$$

Thus, the answers to the task are as follows:

$$\frac{S}{S_1} = \frac{Q}{Q_1} = 2; \frac{S}{Q} = \frac{4}{\pi}.$$

Similarly, the task will be solved with the same condition, but with the modified questions to the task: *Find the ratio of the sum of the perimeters of all squares P (the sum of the lengths of all circles C) to the perimeter of the largest square P_1 (the length of the greatest circle C_1) (or: find the ratio of the sum of the perimeters of all squares to the sum of the lengths of all circles).* Obviously, in this case we have:

$$P_1 = 4a;\ C_1 = \pi a,$$

and each of the sequences (the perimeters of the squares and the lengths of the circles) form diminishing geometric progressions with a denominator of $\frac{\sqrt{2}}{2}$. From here we have answers to this version of the task:

$$\frac{P}{P_1} = \frac{C}{C_1} = 2 + \sqrt{2};\ \frac{P}{C} = \frac{4}{\pi}.$$

We formulate the condition of the task differently. Now we will describe the figures around the given figures.

<u>Task 2.</u> *A circle was described around a square with side a, a square was described around this circle, a circle around this square, etc. until, as a result, received 5 squares and 5 circles. Find the ratio of the sum of the areas of all squares (circles) to the area of the smallest square (circle) (or: find the ratio of the sum of the areas of all squares to the sum of the areas of all circles).*

Already knowing from the previous task, the law of changes in the areas of squares and circles, we use this for our case. So: $S_1 = a^2$; $Q_1 = \frac{\pi a^2}{2}$, denominator of increasing geometric progression $q = 2$. Then we have:

$$S = S_1 + S_2 + \cdots + S_5 = \frac{a^2(2^5 - 1)}{2 - 1} = 31a^2;$$

$$Q = Q_1 + Q_2 + \cdots + Q_5 = \frac{\frac{\pi a^2}{2}(2^5 - 1)}{2 - 1} = \frac{31}{2}\pi a^2.$$

In this way:

$$\frac{S}{S_1} = \frac{Q}{Q_1} = 31;\ \frac{S}{Q} = \frac{2}{\pi}.$$

Note: The condition of this task can be formulated in general terms, not limited to the five described series. Obviously, for the case of n squares and n circles, the answers to the problem will be:

$$\frac{S}{S_1} = \frac{Q}{Q_1} = 2^n - 1;\ \frac{S}{Q} = \frac{2}{\pi}.$$

Similar to the previous task 1, we can formulate other questions for task 2: *Find the ratio of the sum of the perimeters of all squares (the sum of the lengths of all circles) to the perimeter of the smallest square (length of the smallest circle) (or: find the* *ratio of the sum of the perimeters of all squares to the sum of the lengths of all circles).* We obtain tasks that are solved in a similar way.

The two examples of solving tasks illustrate the task theme of "fitting (describing) into (around) given geometrical figure(s) of the combinations of the other geometrical figures", the model of which is a geometric progression. This task topic in turn generates the task series, where a square can be replaced by a regular triangle, hexagon, pentagon or decagon (in the latter two cases, it will be necessary to use the properties of the golden section), etc. Turning to stereometry, this task series can be continued like this.

Task 3. *In a cube whose side is equal to a, a sphere is inscribed, a cube is inserted into a sphere, etc. Find the ratio of the sum of the volumes of all cubes (spheres) to the volume of the largest cube (largest sphere) (or: find the ratio of the sum of the volumes of all cubes to the sum of the volumes of all spheres).*

The elementary considerations make it possible to find out that the volume of the largest cube is $F_1 = a^3$, and the volume of the greatest sphere is $V_1 = \frac{\pi a^3}{6}$. After calculating the volumes of other figures, we establish that the volumes of both cubes and spheres form the diminishing geometric progressions with the denominator $\frac{1}{3\sqrt{3}}$. Thus, finding the sum of volumes of cubes F and spheres V, we get:

$$\frac{F}{F_1} = \frac{V}{V_1} = \frac{3\sqrt{3}}{3\sqrt{3}-1}; \ \frac{F}{V} = \frac{6}{\pi}.$$

Just as we did with the task 1, we can reformulate the question to the task 3*: Find the ratio of the sum of the areas of surfaces of all cubes S (spheres Q) to the surface of the largest cube S_1 (the greatest sphere Q_1) (or: find the ratio of the sum of the areas of surfaces of all cubes to the sum of the areas of surfaces of all spheres)*. Such a task as a decisive model will also have a geometric progression – this time with a 1/3 denominator. Considering that:

$$S_1 = 6a^2; \ Q_1 = \pi a^2,$$

and finding the sums of the areas of the surfaces of all the cubes and spheres as the sum of the members of an infinite geometric progression, we obtain the following answers to this variant of the task 3:

$$\frac{S}{S_1} = \frac{Q}{Q_1} = \frac{3}{2}; \ \frac{S}{Q} = \frac{6}{\pi}.$$

Similar to the task 2, you can continue the task series with a finite description of a sphere, a cube, again a sphere, etc. around a given cube with side a, calculating the relations between the corresponding volumes of the figures or their surface areas. It is clear that in this case all the tasks will have the same decision model, but the content of students' integrative learning activities can be determined on the basis of the individual or differentiated approaches.

Change the shape of inscribing.

Task 4. *In the angle of 60 ° in the direction to the top there are 5 circles inscribed so that each next circle, starting from the 2nd one, touches the previous one. How many times is the sum of the areas of all five corresponding circles larger than the area of the smallest circle?*

It is clear that all the centers of such circles will lie on the bisector of the angle, and the circles themselves will be homothetic with the homothetic center at the top of the corner. If the homothetic coefficients of all 5 circles are multiples of one number, then the decisive model of the problem will again be a geometric progression. Check it out. By the property of a right-angled triangle, one of the angles of which is 30°, its hypotenuse is twice the size of the cathetus opposite to this angle. Hence, the distance from the top of the angle to the center of the smallest circle is 2 times its radius. Denote the radius of the smallest circle by r, the radius of the next circle x. Given the similarity of right-angled triangles that are formed by the radii of these circles, the segments from the top of the corner to the centers of these circles and the corresponding segments lying on the cathetus of the angle, we obtain the ratio for the corresponding proportional segments (radii of circles refer to are related as the distances from their centers to the top of the angle):

$$\frac{x}{r} = \frac{2r + r + x}{2r}, from\ here\ x = 3r.$$

After a similar check of the ratio of the radii of other circles, we establish that the radii of all the circles form a geometric progression with the denominator 3. Hence, the areas of the circles form a geometric progression with the denominator 9. Thus:

$$\frac{S_1 + S_2 + S_3 + S_4 + S_5}{S_1} = \frac{\frac{\pi r^2(9^5 - 1)}{9 - 1}}{\pi r^2} = 7381.$$

Note 1: The condition of this task can be formulated in general terms, not limited to five circles. Obviously, for the case of n circles, the answer to the task will be:

$$\frac{S_1 + S_2 + \cdots + S_n}{S_1} = \frac{9^n - 1}{8}.$$

Note 2: The task can be solved with other angles (for example, 90 °, 30 °, 36 ° – in the latter case, you can use the properties of the golden section). Generalizing the task, it can be formulated for an arbitrary angle $\alpha\ (0° < \alpha < 180°)$. Then the denominator of the geometric progression, which is formed by the radii of the circles that are inscribed in a corner, will be an expression (this can easily be verified by similar considerations to the previous ones):

$$\frac{1+sin\frac{\alpha}{2}}{1-sin\frac{\alpha}{2}},$$

therefore, the denominator of the geometric progression that the areas of these circles form is the square of this expression. So, for the general case of n circles, the answer to the task will be:

$$\frac{S_1+S_2+\cdots+S_n}{S_1}=\frac{\left(\frac{1+sin\frac{\alpha}{2}}{1-sin\frac{\alpha}{2}}\right)^{2n}-1}{\left(\frac{1+sin\frac{\alpha}{2}}{1-sin\frac{\alpha}{2}}\right)^{2}-1}.$$

Note 3. In a similar way, you can solve a modified task for the task 4 (along with its next options - see Notes 1 and 2), in which the question would be: *How many times is the sum of the lengths of all five corresponding circles greater than the length of the smallest circle?* It is clear that in this task the denominator of the geometric progression, which is formed by the lengths of the circles inscribed in the angle, will be 3. Then the answer to the problem will be:

$$\frac{C_1+C_2+C_3+C_4+C_5}{C_1}=\frac{3^5-1}{3-1}=121.$$

Similarly, similar to the previous generalizations of the task on the ratio of the lengths of circles will be solved.

Note 4. For task 4, it can be transformed into a spatial problem: *The cone, whose axial section has an angle at the vertex of 60 °, starting from the vertex, 5 spheres are inscribed so that each next sphere, starting from the 2nd, touches the previous one. How many times is the sum of the areas of surfaces (volumes) of all five corresponding spheres larger than the surface area (volume) of the smallest sphere?* It is clear that in this case the denominator of the geometric progression, which is formed by the surface areas of the spheres, will be the square of the coefficient of proportionality of their radii (i.e., $3^2 = 9$), and the denominator of the geometric progression, which is formed by the volumes of the spheres, there will be a cube of the proportionality coefficient of their radii (i.e., $3^3 = 27$). Then in the first case the answer to the task will be the same as in the task 4, and in the second case it will look like this:

$$\frac{V_1 + V_2 + V_3 + V_4 + V_5}{V_1} = \frac{27^5 - 1}{27 - 1} = 551\,881\ .$$

For such a variant of the task topic, all the transformations of its conditions are possible, similar to those described in notes 1 and 2.

Change the condition of the task 4 to:

<u>*Task 5*</u>*. An infinite number of circles are inscribed in the angle of 60 ° to the top. Every next circle, starting from the 2nd, concerns the previous one. How many times is the sum of the areas of all circles larger than the area of the largest circle?*

Repeating almost literally the course of solving the task 4, we conclude that the sequence of radii of the circles forms a decreasing geometric progression with the denominator 1/3, and therefore the sequence of the areas of the corresponding circles will form a decreasing geometric progression, but with the denominator 1/9. Let the first circle inscribed in a corner will have a radius r. Then, after appropriate calculations, we get the answer to the task:

$$\frac{S_1 + S_2 + \cdots}{S_1} = \frac{\pi r^2}{1 - \frac{1}{9}} : \pi r^2 = \frac{9}{8}.$$

Note that for such an option of setting the topic of the task (task 5) various transformations of its conditions are possible, that are similar to those described in notes 2-4.

Thus, the methodological conditions for the use of the solution and the study of the "task series" generated by a given task topic are:

1. The formation of an integrated image of the task series occurs in the process of detailed analysis and comparison of signs and characteristics of the individual components of the object of study.

2. The choice of the volume of the integrated image of the task series is carried out by taking into account the overall goal of organizing the student learning activities and depends only on the teacher's planning for the possible (or necessary) breadth of the field of opportunities for the student learning activities.

3. When forming an integrated image of the task series, the teacher organizes the process of mentally combining the components of the integrated image according to their essential features. In the process of planning and preparing the formation of an integrated image, the

components of the integrated image are distributed into interrelated classes according to the most essential features according to their similarity; in the process of the direct formation of an integrated image, systematization takes place – the integration of the classes of the components of the integrated image into a single integrity with the subsequent synthesis of new knowledge.

During the conduction of the pedagogical experiment, we used integrated images of the task series in organizing these types of educational work with students in grades 9–11: a) the individualization of the educational activities of students through offering various options for assignments; b) the differentiation of the students' learning activities by the suggestion of the tasks of varying complexity; c) the generalization and systematization of the students' knowledge and skills; d) the creation of the problem situations, the contents of which are integrated images of task series; e) the students' work on the compilation of new tasks that are elements of the volume of the integrated image of the task series; f) an explanation of the new material.

The experimental methodology provided that the control and experimental groups had a sufficient sample that would be representative. The minimum required number of students in groups with an error of $a_0 = 4{,}5\ \%$ and statistical reliability $P = 0{,}95\ (t = 1{,}98)$ is determined by the formula $n = \frac{t^2 P(1-P)}{{a_0}^2}$ and amounted to 92 students (Sachs, 1982). In our conditions pedagogical experiment was conducted with pupils of the 9th (age group 15 years) and 11 classes (age group 17 years) and covered 230 students (113 students in control classes and 117 students in experimental classes) during two school years (Table 1) of two schools in the city of Kropyvnytskyi. The selection was carried out by the method of serial selection. The study was approved by the parent committees of the respective classes; in addition, each of the participants in the experiment was informed about the possibility of failure without any consequences for its status and gave its written consent to participate in the study. The developers of the experimental study were aware of the social responsibility for their actions and were interested in the objectivity of the results of the experiment.

Table 1. The distribution of participants in the pedagogical experiment

Years of the experiment	Control classes		Experimental classes		Total
	Grade 9	Grade 11	Grade 9	Grade 11	
2016/2017	26	31	28	27	112
2017/2018	29	27	30	32	118
Total	55	58	58	59	230

An experiment with each class was conducted during one academic year. At the first stage, the ascertaining part of the experimental work was carried out in the form of students writing a test, that checked the level of students' ability to solve equations, inequalities and their systems, solve problems of studying the properties of elementary functions, solve textual tasks in an algebraic way, and geometric tasks of determining metric properties of planimetric and spatial figures (the work was estimated on a 100-point scale). The results of the tests of each class were defined as the average scores obtained by all the students in the class.

At the second stage, a formative experiment was carried out – the students of the control classes worked according to the usual program. In the process of teaching students of the experimental classes, for the implementation of the above types of the educational work (see points (a) - (f)), task series were used, their models of solution were the geometric and arithmetic progressions, fractional rational equations, systems of equations and inequalities, elementary functions, and also such at which decision the graphic method was used, the method of coordinates, the vector method, the method of introducing an additional angle (an example of such work with one task series, the decisive model of which was geometric progression disclosed in the main part of the article). The formative part of the experimental work was completed with the final diagnosis in which the "increase" of the level of formation of the students of the control and experimental classes of skills to solve mathematical tasks similar to those presented at the ascertaining stage of the experiment was evaluated. Note that the complexity of the tasks presented was higher than before, but the students of the same age (of both of the control and the experimental classes) were given the same tasks. The results of the ascertaining part of the experiment and the final diagnosis after the forming experiment are shown in Fig. 1 (for each class, the arithmetic average is the result of the test in a 100-point assessment system; the symbols of the classes are defined as: for example, 1-K9 is the first year of the experiment, control is the 9th class, 2-E11 is the second year of the experiment, experimental 11th grade). To verify the effectiveness of the proposed methodology, we used the two-sample t-test for independent student samples (Glass, J., Stanley, J., 1976), further considering two samples: sample X - participants in the control group, sample Y - participants in the experimental group. According to the main hypothesis H_0, the proposed technique did not affect the increase in the level of students in the experimental class of skills in solving problems in mathematics. According to the alternative hypothesis H_α, the proposed methodological conditions for using the solution and studying the "set of problems" generated by a given task topic are methodologically expedient. To test the null hypothesis, we found the critical value of the criterion t by the significance level α = 0.05 $t_{кр} = 1{,}98$. The observed value of the criterion $t_{наб} = 3{,}21$. Since $t_{наб} > t_{кр}$, we rejected the main hypothesis H_α and adopted an alternative H_α. At the same time, the statistical results of the training activities of the control group participants throughout the experiment were within the statistical error $(t_{наб} = 2{,}01)$ and slightly improved taking into account the influence of the traditional organization of the educational process, as well as the influence of random factors on the quality of the control work.

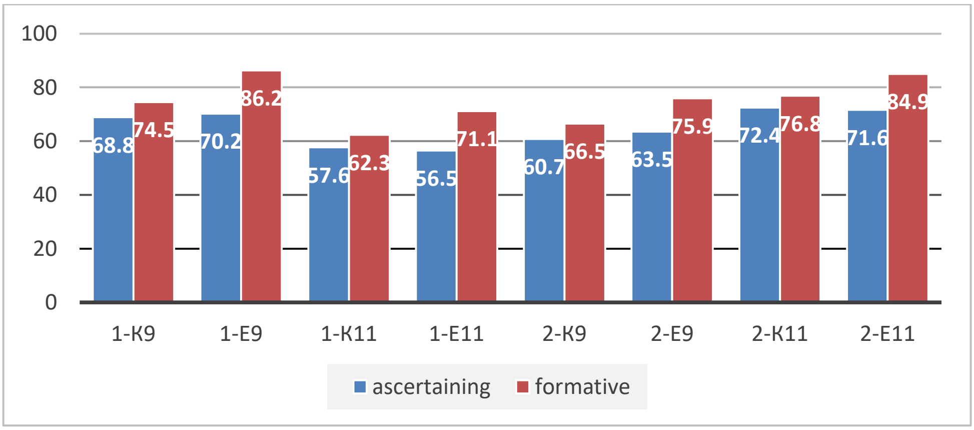

Figure 1: **The results of experimental work**

Discussion

The statistical results of the experimental work showed that the average "increase" in the level of formation of students in the control classes of skills to solve mathematical tasks was 8.02%, and the experimental classes – 21.68%. The data obtained during the experiment have the property of stability between two groups of classes, since the coefficient of variation of the "gains" of the level of formation of the students in the control and experimental skill classes to solve problems does not exceed 18%. Thus, the data obtained indicate that the suggested methodological conditions to use the solution and research of the "task series" generated by a given task topic are methodically expedient in the context of the developing students' skills to implement the integrative activity during the productive operation with the mathematical material.

In the context of the continuation of the study, the stability of the existing level of skills in students to solve mathematical tasks using the proposed methodological conditions should be studied. It is promising to study the methodological features of using an integrated image of a method for solving a task in the process of teaching students. In this way, we understand the holistic structure of skills and abilities that a student (subject of study) needs to own in order to study how to solve a task.

Conclusions

The authors of the article studied the process of working on a task topic, where the approach to the formulation of the conditions of a specific task varied depending on the level (reproductive, productive, creative) skills necessary to solve the task. As a result, the methodological feasibility of using an integrated image of the task theme was investigated. In such a way we understand the integral structure of knowledge and skills that should possess a student (a subject of study) to study "task series" generated tasks theme for their solutions and research. In the course of the experimental work, methodological conditions were determined to ensure the appropriateness of the usage of such an integrated image: a detailed analysis and comparison of the characteristics and characteristics of the individual components of the object of study; the choice of the volume of the integrated image, taking into account the overall goal of organizing student learning activities; organizing the process of mentally combining the components of an integrated image with their essential features and creating a single integrity followed by the synthesis of the new knowledge. The effectiveness of these methodological conditions was confirmed by the experimental data.

The study suggests the methodological conditions of using the task series to be productive in order to form an integrated image of a task topic.

It is advisable to use this methodology for the preparation of the lessons for generalizing and systematizing students' knowledge and skills, when planning and conducting lessons for developing or applying knowledge and skills, and when involving students in developing a system of educational tasks or sets of tasks for measuring the learning achievements.The planning for the formation of an integrated image of the task topic should be carefully done with the use of the first the analysis of the components of the integrated image, then a detailed comparison and comparison of these components in order to further mentally combine the components of the integrated image of the task theme according to their essential features, distribute the components of the integrated image into interrelated classes and further merge not the individual components of the integrated image, but their classes. Then the result of this activity will be the synthesis of new knowledge – the links between the obtained classes of the components and the new synthesized subjects of the training components. As a result, the formation of an integrated image of the task topic chosen by the subject of research is provided under such conditions. It should ensure the formation of students' knowledge and skills of integrative mathematical activity.

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